

Day 0 Teacher Journal

Planning

In talking, in thinking about the mathematical intent, for me, I see this sequence is building toward developing a strong notion in these students of the relationship between sum and average or total and average. I think if you work with students at this particular age level, you'll find that without a doubt, the large proportion of them are able to calculate the average when they're given a set of numbers. That's not a problematic situation for them. I think if you ask them to then go backwards and try to unpack what it is they know when they know the average, that is very problematic. In other words, they're not sure what the relationship between the average and the data set is. For them they have a modal, tendency, a modal idea of average. They think the average should be located towards the center of the data and if you have an outlier or an extreme value and it pulls the average away from the center, sometimes that's problematic. So I'd like to really get them to wrestle with how to unpack the algorithm for finding the average and what the relationship of the pieces of that are with actually the data set.

Another piece of this lesson sequence that I see as being extremely important is the students' eventual development of systems for then going back and reflecting on larger data sets. In particular, I would think that in the first couple of activities that they do in the sequence they'll probably look at some data sets that they, that are either, that they have ranked, or that they come up with a way to rank and then thinking about how they might condense those or combine that information to make a decision or to make a judgement. Initially, I will see them reasoning about procedures that might be reasonable in those task situations. What I'm hoping to do is to be able to shift the level of mathematics in the sequence so that ultimately the students are able to step back from the individual or the specific data sets and think generally or globally about a system that might then work for any particular data set that they are given.

A third piece of this is the role of symbolizing. As this is a pre-algebra class, the students have had some experience in initial symbolizing with respect to variables. And I would think that by taking their systems and then thinking about a way to generalize those (i.e., to symbolize them), that there would be a nice link between the importance of the power of symbols and the development of their systems. It's real important for me when I'm thinking about this lesson sequence, to think about each piece as part of an overall trajectory or part of an overall sequence. In other words, it would be hard for me to say that after the first day I expect these mathematical concepts to be at a certain point or to be developed to a certain degree. I'm looking more as I think about this lesson sequence, what I hope to achieve by the end of the sequence. So I see this as building day after day.

In planning for the lesson, one of the real tensions that I have, and I think that most classroom teachers have, is the balance between trying to script the lesson and then trying to build off of what the students give you. It's so much easier when you're planning a lesson to just script it out and to say, "These are the things I will say and I'll ask the students to do, then they'll do these things, and then this will happen." If you set up open-ended task situations, such as the one these students are going to be working in, you sometimes find yourself at the mercy of the students. I see that as a positive in that what I'm going to try to do is let the mathematics build from the contributions that they make in their investigations. In particular if you look at the first problem situation there are numerous ways that they might go about working this problem, and in having worked with students on this particular sequence before my anticipations would be that they would do a frequency type solution, that they might then just look at the sum of the ranks. Another way is that they might average and then they might also do the reverse sum. Now, if I'm hoping to get those, then in my planning I should anticipate that if I actually do get those from the individual groups, how could I use that to build towards my mathematical agenda? So I would want to think about how to sequence their responses in the whole class discussion so that it builds towards a more sophisticated level of mathematical reasoning. In building toward this, one thing that I have done to allow me the option of getting the notion of sum and average out on the table with the students is that I developed a homework problem in which I particularly make this a salient feature, or I problematize this in that I give them two solutions, one that has been done by finding the sums, one by finding the average.

And I ask them to make some judgements about whether these will always yield the same results. So, I did kind of have a fall back position in that I inserted this problem in the sequence to give me a way to get these issues on the table with the students. However, my hope would be that these would arise naturally from the students' work because I think it's much more powerful when the issues emerge from their own activity as opposed to being introduced by the teacher.

In thinking specifically about the first problem, or the Sneakers problem, one of the things I think is important for me as a teacher to think about is the format of the class and what kind of opportunities I'm going to have for the students to engage in different learning situations. By this I mean, we'll start out with a whole class discussion in which they generate the characteristics or the things that I should look for when I'm buying sneakers. After that they'll go into their small groups to think, come up with a way to rank those. Now, while this is not a mathematical aspect of the lesson, it is a very important piece in that it's their first opportunity to work in small groups. So my job during that time will be to monitor the groups to see that they are interacting in a productive manner. We then come back to whole class as they talk about or share their ranked lists. These ranked lists then become the focus of their next small group investigation, as they come up with a way to combine these lists and then we end by sharing these again in whole class. So there's a nice constant shift between the two different formats of small group and whole class. I think this is a nice feature of this problem because students at this age have such a short attention span that this kind of helps keep them engaged in the problem.

Facilitating

In thinking about the norms for participation I would have to argue that in problem situations such as the one that I'm hoping to do over the next few days that the students' ability to engage, to explain, and justify, to work productively in groups are critical to the effectiveness of this problem sequence. So, as the students are working in their groups, I will have a dual role. One of those will be to actually monitor their activities, so that I have a sense of what the individual groups have done so that I could then think about how I might structure the subsequent whole class discussion. My second job will be to monitor the norms for group work. I intend to make this an explicit topic of discussion prior to their working in groups. In other words, to talk about what their role is, what their job is when they're working in groups. And then I have to follow up in action as I monitor the groups to ensure that those things are actually happening. I think one thing that sometimes happens for us as teachers is that it's real easy for us to say, "I want everyone to work together. I expect you to all take part in this group" and then when we're monitoring the group sometimes we get distracted by the mathematics and we don't attend to whether the norms are actually in place that we would like to be. So we let that slide. As a result, the students respond to what we do, not to what we say. And so you have the possibility of a math authority or simply a dominant personality starting to take over a group and it no longer works productively together.

Another piece that I think is extremely important is that it'll be important for the students to learn how to present arguments or to present their, the ways that they have worked out the problems and ultimately their systems when we're having whole class discussions. One thing that I have found that's typically true of students is that when they're working in their groups, many times they'll come to a clear understanding themselves about the way, or the procedure, or the system that they developed for solving a particular task. Then when it comes time to share that with the whole class, they're very cryptic in their explanations because they assume that the people in the room have just as much information about the system as they do. So, it's going to be important for me to push the students to explain at a level so that all the students in the class understand their system. There won't be any possibility for us to initiate, or for the students to experience, a shift in the level of discourse, such that we start reflecting on discussing the similarities and differences of their systems, unless they are clear about what these individual systems are. So it'll be, that'll be one of my primary goals, is to help the students learn to develop ways to give clear justifications and explanations.

One thing I really like about this problem sequence, or problem sequence of this nature, is that I think in these type situations all of the students in the classroom have a way to engage. In particular, if you look

at the first problem of the sequence, the Sneakers problem, once we have, the students have generated the different rank order lists, I believe all the students in the class would have a way to think about how they might combine those into one list. Now this might be something as simple as just doing a frequency, like which one is in the first slot the most often or it might be something much more sophisticated like finding the average or doing the reverse order sums. Nonetheless, they have an entree into the whole class discussion because they've had an opportunity to think about how they could combine or accumulate those into a single list. The fact that they've done this then gives them an opportunity to engage in a conversation which might then support shifts in their mathematical thinking about how they might then reason about subsequent lists of ranks.

A second aspect of this lesson is thinking about how I might build from the students' contributions to achieve my mathematical agenda. In particular, I need to anticipate how I think the students might go about solving these problems and what that might yield for subsequent whole class discussions. I would venture to say that probably at least one or two of the groups will offer a ranked list that just deals with frequency. In other words, they look across the different lists and see how often a particular item is in a certain rank order. I'm hoping that other groups will give me sums and averages and most likely reverse sums, and by that I mean that number one gets, if there are ten items, number one gets ten points, number two gets nine points, and then they would then sum those. These last three ways should yield the same rank order lists if the mathematics or the arithmetic is done correctly, and that would then be a point of discussion for me to raise the issue of will this always happen, or do we think this is just a coincidence that it happened this particular way. It's in that setting that I hope to begin to tease out the relationship between total and average, and then that I hope will be a recurring theme that will appear as we work the Crime problems on the subsequent days.

Understanding Student Thinking

In thinking about assessing this problem sequence I have to think about it in lots of different ways. In other words, I think that in order for the sequence to develop in an appropriate manner, it will be imperative that I constantly do an ongoing assessment of the students' activity. As the students are working in their groups and I'm monitoring their work that will inform how I structure the whole class discussions. Now, while that's an assessment that I'll have to make in order to act planfully during the discussions, it's also an assessment I'm making of the individual students' mathematical growth. Because if I have, let's say, one group that's working very, in a very sophisticated manner, it gives me the solutions that I need to move the whole class discussions forward, but I feel that none of the other groups are capable of understanding those solutions, then I need to rethink how I'm going to move. So, it will be imperative that I have some sort of a handle on how the students are progressing. To be more specific about individual students' progress, I have two different tasks that are planned within the sequence. The first of these is a homework assignment that will be given after the Sneakers problem on the first day. This will give me more information about the students' current understandings of the notion of average and total. The last piece, or the second piece is an assessment that I will give at the end of the instructional sequence, which will ask them to, in effect, reflect back on the systems that we've discussed in whole class, and modify or adapt one of those systems to work a problem. So I'm hoping that if you take the pieces from the individual assessments and weigh those against what I know about the students' activity in whole class, then I'll be in a position to make an adequate assessment of the students' progress over the lesson sequence.

Mathematical Content and Context

Another important mathematical concept that will arise during this problem sequence is the notion of rate. The problems that are posed on day two and three are grounded in the notion of rate. The crime statistics from the cities are actually murder rates, and so forth. So, it will be important for the students to

have an understanding of rate such that they can engage in the problem situation. As a result, while I was in the classroom last week, I posed some problems to the students about rate. I posed a problem in which the students were doing a newspaper collection campaign and they were doing a contest against their rival school. And I talked to them about the concept of, if we looked at the total number of pounds that each school collected but the two schools had different numbers of students, how could we come up with a way that we could make an equitable comparison about how the two schools did. And the students came up with the idea of looking at the number of pounds collected per student. When they did this, I simply introduced to them that this is what we call a rate, and that rates are often used as a way to equalize and compare sets that are typically not equal. I think this is important that they have some introductory notion of rate in order to be able to do these problems. However, I'm not at all of the belief that they have to be fully grounded in rate in order to work these problems. I think rate is an extremely complex concept that needs to be built through a process of problems and activities that the students engage in through the middle grades. It's a critical aspect of their later mathematics and if we start to build the ground work now by introducing them to problems where maybe rate is not the focus of the activity, the mathematical focus, but it is an aspect of it, then that can only serve to build a strong foundation for this concept.