Mathematical Explorations

(1) Over the course of this lesson sequence, the students and the teacher develop an argument that the ranks of a list of sums are equal to the ranks of a list of averages. How would you state this idea in more precise mathematical language? How would you express this idea symbolically? How would you prove your statement? [See related work in Issues Matrix, Day 2 Class, Student Thinking, Sum & Average - #62 & #68, Day 3 Class, Student Thinking, Sum & Average - #80.]

(2) In another class where this lesson sequence was taught, one group of students used what they called a "reverse ranking" strategy. In their system, they assigned the highest ranking attribute on each list a value of 8 instead of 1. Then they summed these reversed values and used the sum in finding a final ranked list of the data.

> (a) How will the results of this "reverse ranking" system compare to the results of the systems that ranked the sums and averages? Write a precise mathematical statement of the relationship and give a proof of your claim. Describe an acceptable mathematical argument you might expect from a 7th or 8th grader.
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> (b) What is the relationship (if any) between the summed values found using the "reverse ranking" system and the summed values found by the "summing" or "totaling" system?

(3) At one point in this lesson sequence, Nadira suggests that they should count the total violent crime rate twice. The teacher replies that even counting it twice doesn't really change its contribution to the total and suggests that they don't focus on the total property crime rate but just on the total violent crime rate. Rob ignores the suggestion and argues that they should cut the total property crime rate by 2. This discussion results in the expression [TVC*2]+[TPC^2]. Can you find an algebraic expression using only the total violent crime rate (leaving TPC unchanged as the teacher suggested) that would give an equivalent ranking? Justify your answer. [See related work in Issues Matrix, Day 4 Class, Student Thinking, Weighted Ranks - #88]

(4) In the Sneakers Problem, the students need to combine the 6 group lists into a single class list; the rationale for generating an aggregated list was to help the teacher in making a tennis shoes purchase. What information is lost when you create an aggregated list? What information is gained? When would you use an aggregated list like the ones the students created in this lesson? [See related work in Issues Matrix, Day 1 Class, Facilitating, Transitions - #49, Student Thinking, Questions - #59, Mathematical Content, Context - #48, Sum & Average - #52 & #55, Frequency - #50]

(5) Developing number sense is a critical goal for elementary and middle school mathematics. The calculation of the arithmetic mean is done by dividing the sum by a

whole number. This mathematics could be extended to consider other operations. Would the resulting lists be in the same order as the "sum" list if you divided by a fraction or by a negative number? Added a constant to the sum? Subtracted a constant from the sum? Or multiplied the sum by a constant? Why or why not?

(6) In ranking the lists for the sneaker problem on Day 2, the teacher comments on the "loss of proportionality" in the ranked lists. What does she mean by this? Describe how you would develop this concept further with a group of eighth grade students.