MAT 296 - Calculus II

TA: Sum as n goes from 0 to infinity, x minus 2 to the n over n. What's the center?

Student: Two.

TA: Two. Why is the center 2?

Student: Because of when it's equal to 0.

TA: Yeah, taking x minus 2 to the n, when is that equal to 0? What makes my term 0? Two. So, x equals 2 is the center. What is the radius? How do I find the radius? Uh, like Katie said, uh, absolute ratio test. The limit as n goes to infinity, absolute value of an plus 1 over an, limit as n goes to infinity, absolute value of x minus 2 to the n, over n plus 1. And over n plus 1 over x minus 2 to the n over n. And what do I get when I simplify?

Student: x minus 2 times n over n plus 1.

TA: Okay. So I get limit as n goes to infinity. This x minus 2 to the n will cancel with most of the copies of x minus 2 in the numerator. So you have absolute value of x minus 2 times n over n plus 1. And what is that limit?

Student: The absolute value of x minus 2.

TA: Yes, when I take the limit this term goes to 1 so I'm left with absolute value of x minus 2. And again absolute ratio test guarantees convergence when my limit is less than 1. So, when is this less than 1? What's my radius of convergence?

Student: Two.

TA: My center is 2.

Student: One.

TA: The radius of convergence is 1. Um, and what's the interval of convergence? When is x, absolute value of x minus 2 less than 1?

Student: Between 1 and 3?

TA: Between 1 and 3. And, again, I don't know if I include 1 or 3 yet, I need to check the endpoints individually. So, how do I find the radius? I get it into a form, just absolute value of a single multiple of x. There's no coefficient of x other than 1. x has a coefficient of 1 and then it's less than the radius. And I assume you can solve absolute values, this is a calc II class. So, what happens at the endpoints? What happens at x equals 1? What series do I have at x equals 1?

Student: Alternating harmonic.

TA: It's the alternating harmonic. Does the alternating, ehh, does the alternating harmonic series converge or diverge?

Student: Converge.

TA: It converges. How so? It's an alternating series, so is it conditional or absolute convergence? Conditional, does the alternating series converge conditionally or doe sit converge absolutely?

Student: Absolutely?

TA: Ah, yeah, guess not. Um, if I take absolute values of the alternating harmonic series, make all the terms positive. What series do I get?

Student: Harmonic.

TA: The harmonic series, good. Does the harm, does the harmonic series converge or diverge?

Student: Diverges.

TA: It diverges. So, I have the alternating harmonic series. It converges, so 1 is included, the series, the power series does converge at x equals 1. But if I take everything absolute value, it diverges. So it's conditional convergence. Ah, conditional, c-o-n-v-e-r-g-e. So it converges conditionally. It has conditional convergence at x equals 1. How about at x equals 3? What series do I have if x equals 3?

Student: Harmonic series.

TA: The harmonic series. And as Joe just said, the harmonic series diverges. So, at x equals 3, this power series does not converge. So my interval of convergence is closed 1 to open 3. And I should see it diverges at x equals 3. So are there questions on problem 5? Okay, problem 6. And the reason I'm going over problems 5 and 6 is because you should expect problems like these on the final exam. Okay? These are good questions for a final and will cover up through, in fact the final will cover up through the end of chapter 10. Okay? So, problem number 6. I give you two power series and I apologize greatly, um, one of the power series that I gave you is not quite right. Did anyone pick that up? Oh. What's the power series for e to the x?

Student: Isn't it x squared over 2?

TA: It's x squared over 2.

Student: Over 2 factorial?

TA: Over 2 factorial, yes. Um, fortunately for me the only time that that made a difference in peoples' answers was either if they got it completely right or if they missed some other things as well. Ah, so that didn't really effect point values which was lucky for me because typos on exams can change point values students would get. So, e to the x is 1 plus x plus x squared over 2 factorial plus x cubed over 3 factorial, plus x to the 4th over 4 factorial plus and so on. Um, I go up to the 7th term there, fine. Tan inverse of x is x minus x cubed over 3 plus x to the 5th over 5 minus x to the 7th over 7 and so on. For 1, sorry negative 1 less than x less than 1. The radius of convergence for this power series is 1. It's centered at 0 and it does not include the endpoints. Okay? Whereas the radius of convergence for e to the x is infinite. It's the whole real line. So for tan inverse of x there actually is a significance of convergence that you may have to worry about. Um, although no one picked that up on part b, I believe. Oh well. Ah, so part a. Find the first 4 non-zero terms the power series expansion for x e to the 2 x. How would I do that? Katie?

Student: For every x and e to the x, you put in 2x.

TA: Okay.

Student: And then you'd multiply each of those by x.

TA: Right. I have, so what Katie said is that I have x times this series except wherever I see an x I put in 2x. Here is my power, there's my exponent, so I plug that in. So, 1 plus 2x plus, what's my next term?

Student: 4x squared.

TA: 4x squared. So, it's the parenthesis, 2x squared. So, the 2 is getting squared as well. Many people got, I think, like 6 or 7 points on this problem, I think, because they did not square and cube this 2. 2x plus 2 x squared plus either 2x cubed or 4x cubed depending on how it's, this was the way that it was written according to my typo because I did not have that divide by 2 there. But this is what it really should be. I apologize for that. And then 8 6 is 4/3 x cubed times x is x to the 4th. So, that's a. Do, are there any questions about part a?

MAT 296 Calculus II

TA: Alright. Now, I'd actually like to do 10.8 before we do 10.7 if that's all cool with you all. Is that alright?

Students: Yeah.

TA: Yeah?

Student: On like the first few problems where they said like, there was like a first, it was like 8, 8, 1, 5.

TA: Yeah.

Student: I don't understand how they like divide, multiply like infinite...

TA: We talked about that briefly and basically what we said is that you just have to collect up the terms in the right power. So, I'd like to actually do one of these. Number 3.

Student: Like 3 I can sort of see, but like 1, I have no clue on.

TA: Yeah, um, 1 I wouldn't do by dividing. One if I were you, I'd do by taking derivatives. And that actually, the taking derivatives actually writing out the McLoren series and the Taylor's series. That is what I think is more important than being able to divide and multiply big power series, but multiplying makes more sense, so we could look at number 3 because that was a product of e to the x and sin x. We can look at that one real quick just to get an idea what's going on. Did you, did you get 3 or did you have an idea about 3?

Student: Yeah, you try and FOIL it out, but...

TA: And you FOIL it out and you figure out what terms give you what degree terms and you can pretty much figure out what the terms are going to be because they're aren't that many possibilities of, of products of those terms getting up to degree 5. So you really don't have that much work to do. So, if we write this out real quick. e to the x we said was, ah, 1 plus x plus x squared over 2 factorial plus x cubed over 3 factorial, x 4 over 4 factorial, x 5 over 5 factorial plus so on and so forth. I stopped at 5 because they asked me for terms up to degree 5. So, I'm to list all the terms of this up to degree 5 and I know I want to list sin x. Sin x? Is that what they said? Sin x up to degree 5. So, this was x minus x cubed over 3 factorial plus x 5 over 5 factorial minus so on and so forth. So then you just have to figure out when I multiply these things, these two things together which terms give me terms of degree 5 or less. Well, x times all of these guys up to degree 4 gives me terms of 5 or less. So I want to do x times 1 plus x plus x squared over 2 plus x cubed over 6 plus x 4 over 24. Or something. Sure. Does that make sense? If I multiply this x by the next term I get a degree 6 term. They only wanted up to degree 5. And then

I can take this negative x cubed and multiply that by 1, 2, these 3. Because if I multiply it by the next one I get a degree 6 term. So, I guess minus x cubed over 3 factorial times 1 plus x plus x squared over 2. And then plus the degree 5 term times just 1. Is that the entire product of e to the x sin x? No. There's a whole bunch of other stuff that comes after it. But these are all the possible terms that have degree 5 or less. Because the only thing I didn't multiply was x 5 times the rest of these guys which all have degree bigger than 5 or any later terms times all of these guys, which all have degrees bigger than 5. So you multiply these all out, collect up terms, you'll get the answer in the back of the book. For division you do kind of a similar thing, with long division. I didn't really want to get into it. I prefer if you did number 1, you did not do it by dividing sine by cosine. I think that's the harder way to do that problem., I would do it by taking the sum of the derivatives of the tangent because that's really what I'm going to be asking for on the tests and quizzes from now on which amounts to today's test and the final. So, it's like a hint or something.