MAT 295 - Calculus I

TA: Okay, I think we'll start off here with two problems. We may have time for more, but if not we'll talk more about reviewing limits, um, tomorrow if there's time or Monday if we don't get through these all. So, let's look at the limit as x goes to 6. And I guess what I didn't mention is that what we're going to do for reviewing these, what I'd like you to do is find one or two other people to work with on this. Okay? So we're going to work through these problems and then we'll go over them as a group and see if we can remember any of this stuff. Okay? So there's the first one, here's the second one. Okay. So we want to find one or two people to work with on this. My guess is that you'll probably find that talking to other people will hopefully will jog your memory as to how we do these. Do you have a question already?

Student: Yeah. Do we plug in like a different number for here?

TA: We're not looking at the limit from the right. Okay?

Student: Yeah, but if you plug in 6, it keeps on going...

TA: Yeah. What else happens, what happens in the numerator?

Student: This becomes 0.

TA: The denominator becomes 0, but it also matters what happens in the numerator, right? I think, oh, I think what you're thinking, you're thinking, when we did those, um, those right and left hand limits when we were talking about vertical asymptotes, if we had a non-zero constant over zero, we said that it was infinite. Right? Is that what you're thinking?

Student: Yeah. I got it.

TA: Whoa, you're fast.

Student: But what do I do then for x?

TA: What happens if I plug in 6 here into the numerator? Into the whole numerator.

Student: Oh.

TA: Alright. So I've got 0 over 0 which, what does that tell me?

Student: [inaudible]

TA: So, that's not, that's different than that other case where you were thinking of where we had something like 5 over 0. If it's like 5 over 0, then we've got some sort of infinite limit. If we have 0 over 0, what happens?

Student: Um...

TA: What happens if it's 0 over 0? Do you remember?

Student: It's undefined.

TA: It's, but 5 over 0 is undefined also. What's the difference between a case where we plug it in and we get 5 over 0 and a case where we get 0 over 0?

Student: 5 over 0 is undefined. They're the same.

TA: Are they the same. If I...

Student: The difference is you get like some number.

TA: Some number?

Student: That's when you get a number.

TA: Okay, you might get a number and why would that be? You might get a, a non-infinite limit is what you're saying. Is that what you're saying?

Student: Yeah.

TA: Right? Because that means there's some factor in common with the numerator and the denominator.

Student: So, I have to divide by the, the, oh...

TA: You're thinking of limit to infinity. We're going to limit to 6. Yeah, it's been a while since we did these. And you're thinking of the things that we talked about more recently with, with asymptotes, but I think if you talk to Michelle, she's got an idea here.

Student: Now when you times these together doesn't it, does it become positive or is this a negative?

TA: If I had 4 minus x and 4 plus x and I multiplied them together what would I get?

Student: Oh, 0. No, 16.

TA: Can I borrow you're pencil? If I had...

Student: It'd just be 16x plus x, right? Oh.

TA: What does that equal?

Student: Okay it'd be x squared plus, I don't know if it's minus or plus, so it's like plus 16.

TA: Ah, 16 minus x squared?

Student: Yeah. Pretty much.

TA: Right? So, what do I get here?

Student: Wouldn't these just be, well, so...

TA: So instead of those being x...

Student: Oh.

TA: Yeah, so instead of those being x's, they're square root of x's, which means when I square them?

Student: It's just times x.

TA: Oh, is that, that's what you were saying.

Student: Yeah.

TA: Sorry.

Student: Basically, is it positive or negative? It's negative.

TA: Mm-hm.

Student: We got the first one, but...

Student: This one, if you put like 6 in it's going to come out...

TA: You get 0 over 0. So, what are our techniques to deal with that?

Student: Is that the one that we use this function?

TA: Um, we did some examples sort of like, we did some examples of finding the derivative using the definition that involved square roots and we used the technique that we want to use here. I mean the problem here is if I look at this, I don't know how to factor that because I don't know how to factor that denominator, right? If I get 0 over 0, it tells me that there's got to be something in common with this, but this, it's not clear how I can factor that. Remember when we did a technique where we multiplied by the, I think it's called the conjugate.

Student: Mm-hm.

TA: So, we multiplied numerator and denominator by 4 plus square root of x because that's going to get rid of that square root for me. Try that. Remember that?

Student: Yeah, it's starting to come back.

TA: Are we okay on this one? Okay and what did you do on that second one?

Student: We factored, factored it to 4 minus the square root of x.

TA: Okay. The... hmmm...

Student: It factors that way, right?

TA: Yeah, it does.

Student: Did we do it wrong?

TA: No, no, it, that wasn't how I was intending, how I was thinking that you would approach it. But, yeah, that does work, I think.

Student: What was the way we were supposed to do it?

TA: The way I was intending was that you would multiply numerator and denominator by 4 plus square root of x. But, but that does factor that way. So, okay, you thought of a different way. It just is surprising. So, we're okay here?

Student: Yeah.

TA: No problems?

Student: No.

TA: How are we doing over here folks? We doing okay?

Student: Yeah.

TA: We okay?

Student: Yeah, we got, ah, 8 and 2.

TA: Mm-hm.
TA: Alrighty. So, I know some of you all are done with these already so, I'm looking for two volunteers to put these up on the board. Sean, you got the first one? Okay. Another volunteer? Peter? You got the second one? Okay.

[Some students talking in background]

TA: Thank you. Okay, let's take a look at what they put up here on the board. So, what do we think about this first one? Sean put this one up. Do we have any questions for Sean?

Student: He's right.

TA: You think he's right? Did anybody get anything different? Okay. No questions? The only thing that I would say is that we just want to make sure that we remember when we're canceling here... Right? When we're canceling there we need to make sure that we're not looking at, exactly, at x equals 6. Right? Because we can cancel these provided x is not equal to 6, right? Because that would give us 0 over 0. So, since we're looking at this limit, we're looking at places close to the negatives, err, close to 6, but not actually equal to 6. Alright? What about questions for Peter? You had a question, Sean?

Student: Yeah, how did he just take that 4 plus square root of x?

TA: How did you get that? Let him answer.

Student: Um, it could be wrong. It's 4 times 4 is 16 and then you have 4 times square root of x then divided by negative. And then you do the square root of x times square root of x is just x.

TA: Okay. So does that answer your question or?

Student: Well, I thought if you times that by the reciprocal 4 plus radical x and then it gives you 16 minus x on the bottom and then if you leave it's still factored and you can just cancel the 16 minus x right there.

TA: Okay, so there's, there's actually more than one way to approach this problem. Right? I think some folks approached this a little differently. I mean, so Peter was able to see that this numerator here actually factored in that way. Okay? Do we believe that that actually factors in that way? I've still got a couple people looking a little unsure. Do you believe it? I mean if you think about this, when I multiply this together, I mean and, actually, hopefully, if you did it the other way, you saw that if I multiply these things together I get 16 minus x. Alright? So, we can do this, right, we can factor this right here and we need to make sure when we cancel then that, that x is not equal to 16. Right? If x is equal to 16, I got 0 over 0, alright? So, again, because we're looking at the limit, we don't care what actually happens at 16. We care what happens close to 16. Let's just talk through quickly the other way that we could have approached this. I know some of you approached this a little bit differently. So, some of you started out with a limit as x goes to 16 of 16 minus x over 4 minus square root of x. Okay. And what did you do then? So somebody who didn't approach it that way, Nadia?

Student: Multiply it by 4 and then that conjugate thing.

TA: Yeah, this is one of those techniques that we talked about when, I think we talked about it when we were looking at finding the derivative of things like square root of x. We talked about this kind of a technique where you're multiplying by I believe this is called the conjugate of this. Okay? So, it's sort of, if you look at this, I mean this is sort of what Peter was doing there. Alright? So, I multiply by 4 plus square root of x over 4 plus square root of x and what does that give me?

Student: That and then 4 plus square root of x divided by the square root of x.

TA: Mm-hm. And then we're saying then we can cancel this again provided x is not equal to 16. And in the end we end up with the same thing. Okay? So both of those are ways that we could approach this problem. It depends upon how you see it. Okay? So, if you can see it the way Peter say it, great. If you can see it the way some other folks saw it, also great. Okay? Questions on those two?