MAT 284 - Calculus with Business Applications

TA: Alright, so the first thing, I've got your quizzes from last week and any remaining stuff I have left over. Justin, not here. Brian? Jacqueline? Right here in front of me. Peter? Dan? Justin? Paige? Erin? Alison? Not here. Kevin? Cam? Brenna? Nick? Jen? Rebecca? Leland? Will? Right here in front of me. Greg? Brittney? Mike? Jamal? No. Jenna?

Student: Can I get mine back?

TA: Mm-hm. Victoria? Okay, did I miss anyone? Oh, you're in a different class. Hang on. Just have a seat. You're in a different class, different class. Alright, for those of you whoa re from different classes, you come see me after class, I can get your quizzes back to you. Alright, first of all, just to look at this quiz a little bit. Um, there's a couple of different things. Number 1, you had to solve for what the average cost, when it would be at a minimum. And you were given the cost function, so you had to remember that in order to solve for coast, for average cost, that cost is equal to average cost times quantity. And then, so your average cost is your cost function divided by quantity. Um, and then from there, you had to take your fist derivative and your second derivative and solve for q also. So, you didn't do too bad on it, just be careful. Ah, you'll see that on the other quizzes, if you looked at them or if you're planning on looking at them for the test tomorrow, there's several different versions and different kinds of questions that you can see with the cost function and I will go over one of those later on. Also, our elasticity question down at the bottom, you needed to remember that elasticity times percent change in price gives you the percent change in demand. Ah, and again, there's three different formulas really for the elasticity and I'll look at one that's similar, but a different question that you might see on your test for that, as well. Are there any questions on this quiz? Alright, so before I look at these, does anyone have questions on any homework problems first? I know your test is tomorrow so you might have been looking some over and have any questions. No? Alright, what we'll do is we're going to look at one of the other quizzes where they had some different things going on. Let me just pull a different one out. Hmmm, we'll look at these two. Alright. Um, on your quiz you were given a cost function and asked to solve an average cost via minimum. What you need to remember is with all of these you're looking at the first and second derivative and you're solving for quantity. Um, be careful of what they're asking you for. They might ask you to solve for price. They might ask you to solve, ah, just when it's maximized, and not, just give you a price. They might just ask you, what's the quantity when it's maximized. Um, so for example, there was a problem on one of the other quizzes that said, a local candy store purchases special boxes of candy for Valentine's Day. At a price of 24 dollars each, the store can sell 770 of these boxes. So, at 24 dollars there was 770 boxes and for each 1 dollar increase, they had 35 more boxes. 35 fewer boxes. Alright? And what the question asks is at what price will there revenue from these candy boxes be maximized and there was a hint that said first find the demand equation and you may assume that the candy store is a monopolist. Okay, so the first thing you have to look at is this and this says, hint first use the demand equation. So who can tell me what a demand equation normally looks like? Justin, what's a demand equation normally look

like?

Student: Ah, p equals something where q.

TA: p equals something where q is. Okay, so we have something with q. Alright. We're given some prices, we're given some quantities, so we need to make this look like a demand equation. So, if you remember, we had the point slope form of a line. And I'm sure Professor Lewis looked at one in class that was very similar to this. What you have to look at is think of these points, the 24, 70, think of it as, in terms of a point on a graph and then what's changing your dollar increase and your 35 fewer boxes of candy being sold is the slope of your line. Alright? So what we're going to have is an equation but it's with p's and q's instead of x's and y's that we normally see are lines in. Okay, so we have p and we know that our point slope form should look like. Alright? We know we should have some y coordinate and some x coordinate and then our slope. Alright, so in this case we have p and we should have minus and then our p that we're going to be subtracting from, our p point, our p coordinate, is our 24 dollars. Alright, I'll come back to the slope and then our q, what is our q value for that point?

Student: 770.

TA: 770. So our 770 goes in here. The only problem we have to solve for is what is our slope? And you should remember that slope is your change in y's divided by your change in x's. Alright. And if you, this is our p which is normally y. This is normally our x's. Alright, our m is change in y divided by change in x. So what is our change in y that we're looking at in this case? Yeah?

Student: The average cost.

TA: Well, if we're increasing, we have 24 dollars we're starting from. What's our change that they've told us.

Student: One.

TA: One. We're changing by 1 and is it a positive or negative 1?

Student: Positive.

TA: Positive. Aright, so we have change in y's, we have a positive 1 here. And then we need our change in q, so we're starting out with 770.

Student: Negative 35.

TA: A negative 35. We have 35 fewer. So our slope should be a 1 over a negative 35. Alright, so I'm just going to rewrite this over here. Okay, so we now have the equation p minus 24 equals negative 1 over 35 q minus 77. Alright. But we also have to remember that our demand equation doesn't normally look like this. As Justin told us, it looks like p

equals and then something with q's. Alright? We have it in that slope intercept form rather than point slope form, but that's okay because we know from our working with lines that we can change between different forms. So all we have to do here is multiply some things out. So we have negative 1 over 35 times q. We have negative q over 35 and then we have a negative 1 over 35 times a negative 770. Now we have 770 divided by 35. Alright, but we're still not quite there yet and what do we have to do to get to that? To get the way we want it to look like in that demand equation form. Mike?

Student: Um, add 24 to both sides.

TA: Add 24 to both sides. We want it to look like p equals. So when we add 24 to both sides we have q over negative 35 plus 24 plus 770 over 35. And plug it into your calculator and tell me what you get when you add 24 and 770 over 35. Greg?

Student: 46.

TA: Good. So we should get p equals q over negative 35 plus 46, alright? So we're done with the first part of the equation the problem hinted at us to was to find the demand equation. Our first thing. Now the question is asking for at what price will their revenue from these candy boxes be maximized. So we're looking at revenue to be maximized. So we now have p equals, alright, remember that p is price, q is quantity. They want to know when revenue is maximized, so what's the next thing we need to do? What equation do we have to solve for from here?

Student: Take the derivative?

TA: Um, they want to know when revenue is maximized. What kind of equation should we be looking for?

Student: If you multiply that by q?

TA: Okay, we know that revenue is p times q. Alright? So we have p. To solve for our revenue equation we're going to multiply that whole thing by q. So r equals q divided by negative 35 plus 46 all multiplied by q. Alright, and we can multiply that out. Our revenue is q squared divided by negative 35 plus 46q. Now we're going to do what Jackie suggested. We need to take the derivative. We want to look at when it's maximized, we need to look at the derivative. So we have r, we need to look at r prime. Okay, so solve r prime on your own first. Alright, who can tell me what the derivative of the revenue equation is?

Student: 2 q negative 35 plus 46.

TA: 2 q negative 35 plus 46. Alright? Just those regular derivative rules you need to remember. And it reminds you about them on the review sheet. Those were some of the first things you need to remember. Not only for doing this, but also for when you're taking the integrals. Alright? Because your derivatives go one way, your integrals are

going to take you back the other way, so it's important you know those derivative rules. So we now have our first derivative, alright? Now we're getting similar to what you had on your quiz. So we've solved for our first derivative, now what do we need to do? You can look at your quiz to get an idea. Yeah?

Student: Set it equal to 0.

TA: Set it equal to 0. You want to set 2q divided by a negative 35 plus 46 equal to 0. And why do we set it equal to 0?

Student: Um.

TA: How about it?

Student: We're solving for q.

TA: We're solving for q. We need a value for q. So, we have 2q divided by a negative 35 plus 46 equals 0. We can add this to both sides, so we have 2q divided by 35 equals 46. And when you solve for q, what do you get q equals?

Student: 805.

TA: 805. Did anyone else get 805? Okay, so we have q equals 805. So, what, once we have a q, we're assuming right now that it's maximized when q is equal to 805, but how do we check to make sure this is actually true? What do we have to do now?

Student: Take the second.

TA: We've got to take the second derivative. Alright, so here's our first derivative, we're going to keep going and just take our second derivative at when q is 805 and what do we get when we take the second derivative? This one's an easy one.

Student: 2 over negative 35.

TA: 2 over a negative 35. So we have this constant number. And once we solve this, how can we tell if it's maximized at this point? Jackie? We have to look at is it less than 0, alright? 2 over a negative 35, we do have a negative number, so it's less than 0. That means it's maximized at our point q equals 805. Alright? On your quiz, this is where we stopped. Alright. They just want to know what the quantity was when your profit was, your profit was maximized. This question asks, though, at what price will the revenue from these candy boxes be maximized? So, who can tell me what you should do to solve for the price?

Student: Plugging the quantity in.

TA: Plugging the quantity in to which equation?

Student: That one.

TA: This one over here?

Student: Yeah.

TA: We can plug it in there or we can plug it into the one where we solved for p, so p equals. Alright? Remember in the demand equation, your price equals. So we have p equals. We know our quantity is 805, we're going to plug that in. 805 divided by negative 35 plus 46 and when you solve you should get p is equal to 23 dollars. Alright, so as you can see, it's very similar to what you had to do, but you had to go a little bit further in order to get your demand equation to start out with. You weren't just given an equation and asked to take the derivative of it. Alright? You might also see other ones, uh, were you want to find the maximized profit. And profit is revenue and you've got your cost, revenue minus your cost. And you'll either see you're given the revenue, you could be given p and you have to multiply it by q to solve for your revenue. And the other thing you need to remember, one of the questions gave you the revenue, but then the problem for some people was that they then gave you your fixed cost and your variable cost and you had to create the cost function on your own. So, you need to remember the total costs are equal to what? Who can tell me how we solve for the total cost of a product? Greg?

Student: Fixed costs by variable costs.

TA: Fixed cost plus variable costs okay? So don't be surprised if you get a problem on your test where they give you bits and pieces of information and you have to put them all together on your own to solve. Alright, questions on this first problem that we looked at?

MAT 295 CALCULUS I

TA: Let's look at, um, the second item on our agenda today which is trying to look at finding definite integrals by means of substitution. And this is from section 5.8. Did you have any questions maybe from WebWorks? I think one of the WebWorks, I think the one due today has some problems that involve substitution.

Student: Can you do one of those?

TA: Okay. You want us to use my example?

Student: Yeah.

TA: Okay, first of all, substitution just like for indefinite integrals where we were, where we said, if we are given a function f and it's a composite function. f is a function of another function g of x and then times the derivative of that dx. Then we just apply the power rule on f at this function. So when we talk about indefinite integrals we are now saying we have limits of integration. And so what we are doing is we are just saying this should be the same as if we now sub, make a substitution and we call this a u, so we have to change our limits so that in, instead of having a through b, we have them in terms of the new variable of integration. So you'll end up with, instead of a, it should be, the lower limit should be g at a and the upper limit should be g at b. f of u with the substitution and then du. That's roughly writing it in symbols. Now let's see, um, what an example might look like. Okay, in your, uh, books, 5.8 number 31. We need to find, to evaluate this integral from 0 to 1, x plus 2 divide by x squared plus 4x plus 1 dx. Oh, wait a minute, and this is squared, the denominator is squared. x squared plus 4x plus 1 quantity squared. Find that integral. So the first step is to think of how to substitute this function. We want to replace the variable x by the new variable u so that it looks something close to this. Stephanie?

Student: I was just going to say what I think is equal to u.

TA: You would?

Student: x squared plus 4x plus 1 would be u.

TA: You want this to equal u?

Student: Yeah.

TA: Okay, so in that case you're saying we let our new variable, u, equals x squared plus 4x plus 1. Steven, what did you want to say?

Student: I was going to say that.

TA: Say this? Okay, then what do we do next?

Student: Find du.

TA: Find du. So, we take the derivative, du equals. And what is du?

Student: 2x plus 4.

TA: 2x plus 4.

Student: dx.

TA: dx. Remember we are differentiating u with respect to x so we have du over dx equals 2x plus 4. But then since we just want to use du alone our new integrand then we are multiplying both sides by dx. So we have it that way. Then how do we use this new information? Travis?

Student: ...equals x plus 2x, so you can take out 2x plus 4...

TA: We are saying in the integrand, original, we have x plus 2, so this is multiplied by 2 so you have to take out 2 and then say x plus 2.

Student: And then it's 2 over 2x plus...

TA: dx.

Student: ...1x and then x plus 2x...

TA: 1/2 du equals x plus 2 dx. So, in this case we have x plus 2 dx which is exactly what we have in the original one and the expression in parenthesis we have replaced it with the u, so we are calling it u. So it will be u squared. How about our limits, 0 through 1.

Student: We need to change them.

TA: We'll want to change them to be in terms of u and... Carlos, when x is 0, you just walked in, what is u going to be?

Student: u will be 1.

TA: u will be 1. u is 1 and then when x is 1, what is u?

Student: Two.

TA: u is going to be 2. Are they both correct? Class?

Student: Some aren't right.

TA: Some of them are not right.

Student: That one isn't.

TA: Jessica?

Student: [inaudible]

TA: It should be?

Student: Six.

TA: It should be 6 instead, why?

Student: Umm..

TA: Do you know why? Do you now see why it should be 6 or?

Student: [inaudible]

TA: Okay, I think maybe you are taking it too fast. This is u. This is u. x squared plus 4x plus 1. So, when x is 1, we have 1, 4, 1. That gives us 6. Then we go back to rewrite our integral with the new limits and the new variable, so we have from 1 to 6, x plus 2 dx is du and x squared plus that, that quantity is u squared, okay, x plus 2 dx was 1/2 du. And since 1/2 is a constant, you can write it outside the integral sign. So, once we are here we are back to things that are familiar, so we just apply the power rule and get our integral. And if you like you can rewrite it in index form so that you use negative 2 as the power instead of 1 over u squared. Try to evaluate that integral. Let's see what it is.

Student: [inaudible]

TA: Jessica?

Student: It's the same thing as that.

TA: It is?

Student: [inaudible]

TA: Okay, let me hear what she said first.

Student: Negative 1 over 2u.

TA: Negative 1 over 2u. Is u the denominator or?

Student: Yeah.

TA: Okay, like this?

Student: Right.

TA: Um, is that including...

Student: 1/2.

TA: Have you already taken care of this or we need to?

Student: Yeah.

TA: You have?

Student: Yeah.

TA: Okay. So, so then you're saying from 1 to 6 and Stephanie, what was your question?

Student: Is that right? Is that the right answer?

TA: Is this right?

Student: I thought, I thought you had to take the antiderivative of that. I don't know.

TA: You don't have to be right, you don't have to be wrong. Just tell us what you are thinking and then we'll...

Student: I guess I was thinking of what I was supposed to do...

TA: Mm-hm. What were you doing? That's the question, what were you doing?

Student: I'm not sure.

TA: Okay, maybe explain to us what you did so that one of us can...

Student: I took that and to the negative 1 and then I divided.

TA: You, you took this which is u to the negative 1 divided by?

Student: Negative 1.

TA: Negative 1. And then multiply by 1/2. The 1/2 that we had there so it becomes this and that.

Student: Alright, so let's say I multiplied buy 1/2, is that the same thing as you?

TA: Okay, yeah. So, just that 1/2. Then just plug in the limits. So we'll have negative 1/2 and you can keep 1/2 out if you like so that we'll have 1 over 6 minus 1 over 1.